Easter Algorithms

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What will be the date of Easter Sunday in 2011?

It is surprising that, although Easter is a lunar feast, the date can be calculated precisely. An Easter algorithm is a method for calculating the date on which Easter Sunday falls in any given year according to the Gregorian calendar. We will describe one such algorithm for calculating the date, presented in Knuth (1997, pp. 159-160), and comment on its potential use in the classroom.

In Table 1, we present the algorithm by means of an example, avoiding mathematical jargon as much as possible.

Of course, we can calculate the date of Easter by hand. We can also use a spreadsheet to calculate the date. To do this, we must express the algorithm by a sequence of steps expressed by mathematical formulae.

We can write the algorithm in a language that expresses the steps in preparation for writing code for a computer program, often called "pseudo-code".

But first we need to define the "floor function" as follows. If Y is any number, then $\lfloor Y \rfloor$ (read as "floor Y") is the largest integer less than or equal to Y.

Thus, $\lfloor 5 \rfloor = 5$, $\lfloor 4.9 \rfloor = 4$, and $\lfloor -4.9 \rfloor = -5$. Now we can write the algorithm in pseudo-code.

Let X be any year.

A : = X – 19*$\lfloor$X/19$\rfloor$

B : = A + 1

C : = $\lfloor$X/100$\rfloor$ + 1

D : = $\lfloor$3*C/4$\rfloor$ – 12

E : = $\lfloor$(8*C + 5)/25$\rfloor$ – 5

F : = $\lfloor$5*X/4$\rfloor$ – D – 10

G : = 11*B + 20 + E – D

H : = G – 30*$\lfloor$G/30$\rfloor$

If either (H = 25 and B > 11) or H = 24 then H : = H + 1

I : = 44 – H

If I < 21 then I : = I + 30

J : = (F + I) – 7*$\lfloor$(F + I)/7

K : = I + 7 – J

Step	Example
Let X be any year	X = 2007
Divide X by 19 and let A be the remainder.	A = 12
B = A + 1	B = 13
Divide X by 100, throw away the remainder, add 1, and call the answer C.	C = 21
Multiply C by 3, divide the answer by 4, throw away the remainder, subtract 12, and call the answer D.	D = 3
Multiply C by 8, add 5, divide the total by 25, throw away the remainder, subtract 5, call the answer E.	E = 1
Multiply X by 5, divide by 4, throw away the remainder, subtract D, subtract 10, call the answer F.	F = 2495
Multiply B by 11, add 20, add E, subtract D, call the answer G.	G = 161
Divide G by 30 and call the remainder H. (NB: If either H = 25 and B > 11, or H = 24, you should increase H by 1.)	H = 11
Let I = 44-H. (NB: If I < 21, increase I by 30.)	I = 33
Take F, add I, divide the result by 7, call the remainder J.	J = 1
Let K = I + 7 – J.	K = 39
If K ≤ 31, then Easter Sunday is the K-th day in March. If K > 31, then Easter Sunday is the (K – 31)-th day in April.	In 2007, Easter Sunday is 8 April.

Table 1: An Easter algorithm

In 2011, Easter Sunday will fall on 24 April, which is very late.

If K ≤ 31 then Easter Sunday is the K-th day in March; otherwise Easter Sunday is the (K-31)-th day in April.

This description of the algorithm can be streamlined by introducing the "mod" function. The quantity $A := X - 19*\lfloor X/19 \rfloor$ is simply the remainder when X is divided by 19. One can represent this as $A := X \pmod{19}$. Similarly, the mod function can be used in calculating H and J. Both the "floor" function and the "mod" function are functions in Excel.

This algorithm is only one of many Easter algorithms; see Ng (2003) and Bien (2004) for discussions of the history of this fascinating subject. Mallen (2002) uses an approach based on looking up tables.

Although we take the calendar for granted, there is a long history behind the development of the calendar dating back to Julius Caesar and earlier. For practical purposes, we all use the Gregorian calendar, but other calendars are still used for a variety of religious and cultural purposes. Almost every year, the difference between the Orthodox calendar and the Gregorian calendar is the basis of a news item in the media. This gives an opportunity to discuss Easter algorithms and explore links between mathematics and historical and cultural aspects of our society.

Students who encounter the algorithm at school can talk about it at home and impress the family. It provides a good response to the question "What did you do at school today?"

During their years in secondary school, students do not meet algorithms that are more difficult than the general formula for solving a quadratic equation. Here is an algorithm that is long and impressive, although it does not contain any really difficult steps. If students want to calculate the date of Easter Sunday using Excel, then they will meet the *floor* function, the *mod* function and conditional statements such as "If I < 21 then I : = I+30". This will give students an opportunity to extend their knowledge of mathematics and programming. See Figure 1.

EASTER Calculator - Knuth's Algorithm (Enter Year)

Step	Formula in Excel	
X	D8	2009
A	MOD(D8,19)	14
B	D9 + 1	15
C	FLOOR(D8/100,1)+1	21
D	FLOOR(3*D11/4,1) - 12	3
E	FLOOR((8*D11+5)/25,1)-5	1
F	FLOOR(5*D8/4,1)-D12-10	2498
G	11*D10 + 20 + D13 -D12	183
H	MOD(D15,30)	3
If ((H = 25) and (B > 11)) or (H = 24) then H := H+1	IF(OR(AND(D16=25,D10>11), D16=24),D16+1,D16)	3
I	44 - D17	41
If I < 21 then I := I+30	IF(D18<21,D18+30,D18)	41
J	MOD(D14+D19,7)	5
K	D19+7 - D20	43
If K ≤ 31 then Easter Sunday is the K-th day in March; otherwise Easter Sunday is the (K-31)-th day in April.	IF(D21<=31,D21,D21-31) IF(D21<=31,"March","April")	12 April (Result!)

Figure 1: Using a spreadsheet to calculate Easter

Note

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References and Further Reading

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