Inverse hyperbolic functions:

Function	Inverse Function	Calculations
$\sinh(x)$	$\text{arcsinh}(x) = \log_e\left(x + \sqrt{x^2+1}\right)$	
$\cosh(x)$ Restriction: $x \in [0, \ +\infty)$	$\text{arcosh}(x) = \log_e\left(x + \sqrt{x^2-1}\right)$, Domain: $x \in [1, \ +\infty)$ Range: $[0, \ +\infty)$	Let $x = \dfrac{(e^y + e^{-y})}{2}$ where $y = f^{-1}(x)$ $\Rightarrow (e^y)^2 - 2xe^y + 1 = 0 \qquad \Rightarrow e^y = x \pm \sqrt{x^2-1}$ $\Rightarrow y = \log_e\left(x \pm \sqrt{x^2-1}\right)$. To decide which root to keep, note that $\dfrac{5}{4} \in \text{dom}(f)$ therefore the point $\left(\log_e(2), \ \dfrac{5}{4}\right)$ lies on the graph of *f* therefore the point $\left(\dfrac{5}{4}, \ \log_e(2)\right)$ lies on the graph of $f^{-1}(x)$: $\log_e(2) = \log_e\left(\dfrac{5}{4} \pm \sqrt{\dfrac{25}{16} - 1}\right) = \log_e\left(\dfrac{5}{4} \pm \dfrac{3}{4}\right)$ therefore the negative root solution is rejected: $y = \log_e\left(x + \sqrt{x^2-1}\right)$.
$\tanh(x)$	$\text{arctanh}(x) = \dfrac{1}{2}\log_e\left(\dfrac{1+x}{1-x}\right)$ Domain: $x \in (-1, \ 1)$ Range: R	Let $x = \dfrac{e^y - e^{-y}}{e^y + e^{-y}}$ where $y = f^{-1}(x)$ $\Rightarrow x\left(e^y + e^{-y}\right) = e^y - e^{-y} \qquad \Rightarrow e^y(x-1) + e^{-y}(x+1) = 0$ $\Rightarrow (x-1)\left(e^y\right)^2 + (x+1) = 0 \qquad \Rightarrow \left(e^y\right)^2 = \dfrac{x+1}{1-x} \qquad \Rightarrow e^y = \sqrt{\dfrac{x+1}{1-x}}$ ($e^y > 0$ therefore the negative root is rejected) $\Rightarrow y = \log_e\left(\sqrt{\dfrac{x+1}{1-x}}\right) \qquad = \dfrac{1}{2}\log_e\left(\dfrac{1+x}{1-x}\right)$.