Problem solving for mathematics classes in Australian high schools

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Contents

Chapter 1

The mission

This book is a set of mathematical problems and we learn through solving problems. Only a good knowledge of primary school mathematics is assumed.

Some of the questions are open-ended. There is no single correct answer: for example, "How much does it cost to keep a dog for a year?" In this problem, you make assumptions about the cost of purchasing the dog, the costs associated with dog food, dog bedding, registration costs, vet costs. Will you estimate the cost for the first year, or the cost for subsequent years, or the annual cost averaged over the life-time of the dog? These open-ended problems are mathematical problems that we encounter in our daily life, and you don't find them in mathematics text books.

The book is divided into chapters: arithmetic, geometry, games, and projects. Within each chapter, the problems are not presented in any particular order.

By and large, these problems are technology-free although the internet will be useful for the projects in chapter 5. Otherwise, calculators and computers should not be used in solving them. This will improve your numeracy skills and your ability in problem solving. Working with other students is encouraged. It is important to show the working for wherever possible. You can use a calculator to check your answers.

No need to rush. Enjoy thinking.

Chapter 2

Arithmetic

1. Calculate: (a) 73.45×10 (b) $73.45/10$ (c) $\$43.56 + \$72.95 + \$4.76$

2. Calculate: (a) $73.45 - 53.22$ (b) $67.15 + 15.32 + 18.91$ (c) $15 + 2 \times 4$

3. Calculate: (a) $20 - 2 \times 4$ (b) $15 + 3 \times 4 - 6 \times 3$ (c) $3 \times 10 + 7$

4. Calculate: (a) $\frac{1}{2}$ of 246 (b) $\frac{1}{2}$ of 147 (c) $\frac{1}{2}$ of 5678

5. Calculate: (a) $\frac{1}{4}$ of 244 (b) $\frac{1}{4}$ of 146 (c) $\frac{1}{4}$ of 5678

6. Calculate: (a) 10% of 246 (b) 20% of 147 (c) 15% of 300

7. Convert to fractions. (a) 10% (b) 20% (c) $33\frac{1}{3}\%$ (d) 75%

8. Convert to percentages. (a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$

9. Convert to percentages. (a) $\frac{2}{4}$ (b) $\frac{2}{5}$ (c) $\frac{2}{6}$

10. Write the following numbers in increasing order: 7.564, 4.567, 5.467, 7.654, 6.457

11. Write the following numbers in decreasing order: 6.574, -5.467, 4.465, 7.654, 6.457

12. Write the following numbers in increasing order: 0.05478, 0.00783, 0.7854, 0.00001

13. How much would it cost to keep a dog for a year? Explain your answer and show your working. [7]

14. Is it less expensive to use taxis for travel for a year than to use your own car?

15. An item in a store usually sells for \$240. For this week only, the item is advertised as 10% off the usual price. What is the discounted price?

16. An item in a store usually sells for \$240. For this week only, the item is advertised as 20% off the usual price. What is the discounted price?

17. An item in a store usually sells for \$240. For this week only, the item is advertised as 15% off the usual price. What is the discounted price?

18. An item in a store usually sells for \$240. For this week only, the item is advertised as $33\frac{1}{3}\%$ off the usual price. What is the discounted price?

19. For this week only, all items in a store are advertised as "up to 20% off the usual price". In a few sentences, explain why the meaning of this advertisement is not clear. How could the owner of the store improve the wording of the advertisement?

20. Here is another confusing advertisement. "Buy two DVDs and get one free." Why might this be confusing? How could the wording be improved?

21. Explain how you would estimate the number of people who attended a large anti-government protest in a reasonably sized city. Choose a specific protest and city; you are free to make one up.

22. Explain the meaning of "1 sq. km."

23. The area of mainland Australia is 7,656,127 sq. km. The area of Victoria is 227,038 sq. km. <u>Estimate</u> the proportion of the area of Australia that is occupied by Victoria. Remember: no calculators and show your working.

24. The area of mainland Australia is 7,656,127 sq. km. The area of New South Wales is 801,137 sq. km. <u>Estimate</u> the proportion of the area of Australia that is occupied by New South Wales.

25. During the gold rush in the 19th century, many Chinese people came to Bendigo seeking gold. As they were not permitted to land in Melbourne, they were forced to land in Robe in South Australia and <u>walk</u> to Bendigo. The distance from Robe to Bendigo is about 450 km. Estimate how long it might take a group of people to walk this distance. Give your reasons and show your working.

26. Once I became a multi-millionaire. I visited Iran where the unit of currency is the rial (abbreviated to IRR). The usual abbreviation for "Australian dollar" is AUD. At the airport I exchanged 400 AUD for rials. The current exchange rate is 1 AUD = 29,590.46 IRR. How much would I have received at this rate (ignoring charges imposed by the currency exchange dealers)?

27. In Indonesia, the unit of currency is the rupiah (IDR) and the current exchange rate is 1 AUD = 9,799.90 IDR. Would 400 AUD make me a millionaire in Indonesia (ignoring charges imposed by the currency exchange dealers)?

28. In the USA, the unit of currency is the dollar (USD) and the exchange rate, when I wrote this problem, was 1 AUD = 0.704056 USD. What would I get for my 400 AUD in USD (ignoring charges imposed by the currency exchange dealers)?

29. When leaving the USA to return to Australia, I had 26.50 USD in my pocket. The current exchange rate was 1 USD = 1.42034 AUD. How much would I get for my 26.50 USD in AUD (ignoring charges imposed by the currency exchange dealers)?

30. Some years ago, I used my credit card to buy a chess set, on-line, from the USA for 200 USD when the exchange rate was 1 USD = 1.22 AUD.

    (a) Ignoring transaction fees, how much did I pay for the chess set in AUD?

    (b) It turned out that the supplier of the item could not fill the order and, eventually, gave me a full refund of 200 USD. However, by this time, the exchange rate was 1 USD = 1.32 AUD. Ignoring transaction fees, how much did I get back in AUD?

31. Calculate the following: (a) $20 - 2 \times 4 + 3 \times 3$ (b) $15 + 3 \times 4 - 5 \times 3 + 7 \times 2$ (c) $3 \times 10 + 7 - 4$ (d) $15 \times 2 - 7 \times 2 + \frac{4}{3} - 5 \times 0$.

32. Brackets make mathematical expressions easier to understand. Re-write the expressions in the previous problem using brackets; e.g.

$$\text{(a)}\, 20 - 2 \times 4 + 3 \times 3 = 20 - (2 \times 4) + (3 \times 3).$$

33. Calculate 14×99. Hint: Think about an effective way to multiply by 99.

34. Calculate (a) 23×99 (b) 76×99 (c) 145×99

35. Calculate (a) 23×999 (b) 76×999 (c) 145×999

36. Two numbers multiply together to give 36,000. Give at least five examples of what the numbers might be.

37. Put the following numbers in increasing order: $\frac{1}{2}, \frac{2}{3}, \frac{1}{5}, \frac{1}{3}$.

38. Put the following numbers in decreasing order: $\frac{1}{12}, \frac{1}{2}, \frac{1}{6}, \frac{3}{4}$.

39. Express the number 60 in terms of whole numbers using the operations $+$, $-$, $\times$, and $/$ in at least 5 different ways. (For example, $60 = 2 \times 30$.) Be imaginative.

40. In how many ways can you arrange the letters AB in a line to form different words? (Here we are not talking about real words; for example, we regard AB, BA are two different words.)

41. In how many ways can you arrange the letters ABC in a line to form different words?

42. In how many ways can you arrange the letters ABCD in a line to form different words? Hint: Be methodical.

43. In how many ways can you arrange the letters ABCD in a line to form different words that start with the letter A?

44. In how many ways can you arrange the letters AACD in a line to form different words?

45. In how many ways can you arrange the letters AAAD in a line to form different words?

46. Imagine that Andrew, Barbara, Christopher, and Dianne are at a dinner party sitting at a circular table. How many different arrangements can we have? (Note: If we go clockwise around the table, then ABCD is the same arrangement as DABC.)

47. You have a cube of cheese and a knife. How many straight cuts of the knife do you need to divide the cheese into 27 little cubes?

48. The distance between Earth and the Moon is about 384,400 km. A spacecraft takes about 3 days to travel from Earth to the Moon. What is approximately the average speed of the spacecraft in km/hour?

49. The distance between Earth and Mars is about 54.6 million km. Estimate how long it would take a spacecraft to travel from Earth to Mars if it travelled at the same average speed as in the problem 48.

50. How many whole numbers between 1 and 1000 contain a 3? (For example, the numbers 3 and 73 contain a 3, but the numbers 81 and 90 do not contain a 3.) [5]

51. If I buy some shares on the stock market for $5 per share and sell them for $6 per share, what is my percentage profit? (Ignore transition costs.)

52. If I buy some shares in a company on the stock market for \$0.0005 per share and sell them for \$0.0007 per share, what is my percentage profit? (Ignore transition costs.) What sort of company do you think this was?

53. I own a hotel with infinitely many rooms. The rooms are numbered 1,2,3,... Fortunately for me, business is good and every room is occupied. One day, a car pulls up and a fellow walks in asking for a room. What can I do to accomodate him?

54. We say that a number is divisible by 2, if 2 divides into the number with no remainder.

    Which of these numbers are divisible by 2? 35, 66, 89, 1234, 12345, 70, 320.

    What is the rule?

55. Which of these numbers are divisible by 10? 35, 66, 89, 1234, 12345, 70, 320.

    What is the rule?

56. Which of these numbers are divisible by 5? 35, 66, 89, 1234, 12345, 70, 320.

    What is the rule?

57. A number is divisible by 4 if the number formed by the last two digits is divisible by 4.

    For example, 1798<u>24</u> is divisible by 4 because 24 is divisible by 4.

    However, 1798<u>25</u> is not divisible by 4 because 25 is not divisible by 4.

    Which of these numbers are divisible by 4? 56735, 45266, 18889, 412394751244, 12345, 70000, 1320, 184882949596500472856472854712O.

58. Will there be a summer Olympic Games in the year 4568?

59. A number is divisible by 8 if the number formed by the last three digits is divisible by 8.

    For example, 17985<u>824</u> is divisible by 8 because 824 is divisible by 8.

    However, 17985<u>825</u> is not divisible by 8 because 825 is not divisible by 8.

    Which of the following numbers are divisible by 8? 56136, 45266, 18888, 4123947512440, 12345, 70000, 1320, 789365738572901437846887, 1000000000000000020000000000040.

60. A number is divisible by 3 if the sum of the digits in the number is divisible by 3.

    For example, 45 is divisible by 3 because 4+5=9, and 9 is divisible by 3.

    However, 28 is not divisible by 3 because 2+8=10, and 10 is not divisible by 3.

Which of the following numbers are divisible by 3? 35, 66, 89, 1234, 12345, 70, 320, 1000000000000000002, 10000000000000000020000000000001.

61. A number is divisible by 6 if it is divisible by 3 and divisible by 2. Is 1000000000000000002 divisible by 6?

62. A number is divisible by 9 if the sum of the digits in the number is divisible by 9. Check this rule.

63. The number 34 can be written in words as "thirty-four". Write the following numbers in words: 123; 1,234; 12,345; 123,456; 1,234,567; 12,345,678; 123,456,789.

64. How many days have you been on this earth? (Don't forget leap years.)

65. How many seconds are there in a year which is not a leap year?

66. I want to make a vegetable garden in the shape of a rectangle. I have 60 m of fence for the garden. What might the area of the garden be? What would be the largest area of a garden that I could make with such a fence? [7]

67. A rectangle has an area of 60 cm^2. What might be its perimeter? [7]

68. The area of mainland Tasmania is 68,401 km^2 and, at the time of writing this problem, there were approximately 7.7 billion people if the world. If all the people in the world were put in Tasmania, how much area would we have each? [9]

69. At what times are the hour hand and the minute hand on a clock at right angles to each other?

70. The city council is planning to create a large rose garden in a park. An experienced worker (Exp) and an apprentice (App) are employed to do the job. Exp could complete the job in 4 days working alone, whereas, App could complete the job in 6 days working alone. How many days will it take the two workers to complete the job together?

71. In year 12 of a school there are 270 students. There are 110 students are involved in athletics, 100 students are involved in swimming, and 40 students who are involved in both athletics and swimming. How many students are involved in neither swimming nor athletics?

72. Find a simple method for multiplying by 9 without a calculator.

73. Find a simple method for multiplying by 11 without a calculator.

74. A student writes this: $\frac{34}{6} = 5.4$. Explain why this is not correct. What might the student have been trying to say?

75. On Monday, a shopkeeper sold **30** apples at **3 for $1**, and received **$10** from customers.

    On Tuesday, the shopkeeper sold **30** apples at **2 for $1**, and received **$15** from customers.

    So, over the two days the shopkeeper received a total of **$25** for the 60 apples.

    On Wednesday, the shopkeeper sold **60** apples at **5 for $2**. But the shopkeeper received only **$24** for the 60 apples.

    Where is the missing $1? [2]

76. Here we introduce a new symbol. $5! = 1 \times 2 \times 3 \times 4 \times 5$. We read 5! as "five factorial". Similarly we can define $n!$ for any whole number n.

    (a) Calculate 3!, 4!, 5! and 6!.

    (b) Calculate $\frac{3!}{2!}, \frac{6!}{5!}, \frac{10!}{9!}, \frac{150!}{149!}$.

    (c) Prove that the number of seconds in 6 weeks is 10!.

77. The following diagram is known as Pascal's triangle.

$n = 0$							1						
$n = 1$						1		1					
$n = 2$					1		2		1				
$n = 3$				1		3		3		1			
$n = 4$			1		4		6		4		1		
$n = 5$		1		5		10		10		5		1	
$n = 6$	1		6		15		20		15		6		1

    (a) Study the pattern and fill in the next few rows.

    (b) Find a formula for the sum of the numbers in each row.

    (c) What would be the sum of the numbers in the row for $n = 10$?

    (d) Find some other interesting patterns in this triangle of numbers.

    (e) Share your findings with other students and pool your findings.

78. In the Bible we find a description of the construction of Solomon's temple. The description contains the following passage.

> And he made the Sea of cast bronze, ten cubits from one brim to the other; it was completely round. Its height was five cubits, and a line of thirty cubits measured its circumference. (1 Kings 7:23, NKJV)

From this description, sketch the bronze cast and then find an estimate of π.

79. Suppose that every person has exactly two names, a given name and a family name (e.g. John Smith). How many different set of initials are possible? How many possibilities are there if every person has exactly three names (e.g. John Andrew Smith)? How many possibilities are there if every person has only two or three names. [3]

80. Estimate the surface area of your own body.

81. Here are the requirements for making a butter spongecake (biscuit au buerre) serving 10–12 people. The units are imperial units. What would be the requirements, in metric units (or SI units), for making the cake for 6 people? ([1, p. 712]).

    - A round cake tin, 10 ins in diameter and 2 ins deep
    - 2 oz. butter
    - A $2\frac{1}{2}$-qt. mixing bowl
    - An electric beater
    - 4 oz. castor sugar
    - 4 egg yolks
    - 2 tsp vanilla extract
    - 4 egg whites
    - pinch of salt
    - 2 tbl. castor sugar
    - rubber spatula
    - $3\frac{1}{2}$ oz plain flour

Chapter 3

Geometry

You will need an Oxford Set of Mathematical Instruments, pencil, eraser, and exercise book.

1. Construct a semi-circle as follows.

   Let A and B be points on a line. Draw a circle with centre A and radius equal to the length of AB, but draw only that part of the circle that is above the line. (You may need to extend the line.) Then you have a semi-circle.

2. Construct an equilateral triangle on a given base as follows.

   Let AB be the given base. Draw a circle with centre A and radius equal to the length of AB. Draw another circle with centre B and radius equal to the length of AB. Let E be one of the points of intersection of the two circles. Draw $\triangle ABE$; this is an equilateral triangle. Why?

3. Construct an isosceles triangle that is not an equilateral triangle.

4. Find the mid-point of a given line as follows.

   Let AB be a given line. Draw a circle with centre A and radius equal to the length of AB. Draw another circle with centre B and radius equal to the length of AB. Let E, F be the two points of intersection of the circles. Draw the line EF and let M be the point where EF intersects AB. Then M is the mid-point oil AB. Why?

5. Construct the perpendicular bisector of a given line.

6. Construct a line that is parallel to a given line.

7. Without using a protractor, construct a line that is perpendicular to a given line. Check the size of the angle with a protractor.

8. Construct a square.

9. Construct the smallest circle that contains the square constructed in problem 8.

10. Construct a regular hexagon as follows.

    Draw a circle with centre O and radius r. Draw a radius from O to the circumference and suppose that it cuts the circumference at A. Draw an arc with centre A and radius r and suppose that this arc cuts to circumference at B. Now draw an arc with centre B and radius r and suppose that this arc cuts to circumference at C. Next draw an arc with centre C and radius r and suppose that this arc cuts to circumference at D and so on, until you have six points on the circumference A, B, C, D, E, F. The polygon $ABCDEF$ is a regular hexagon. Why?

11. Bisect a given angle.

12. Without using a protractor, construct an angle that is $45°$. (Hint: Use problems 7, 11) Check the size of the angle with a protractor.

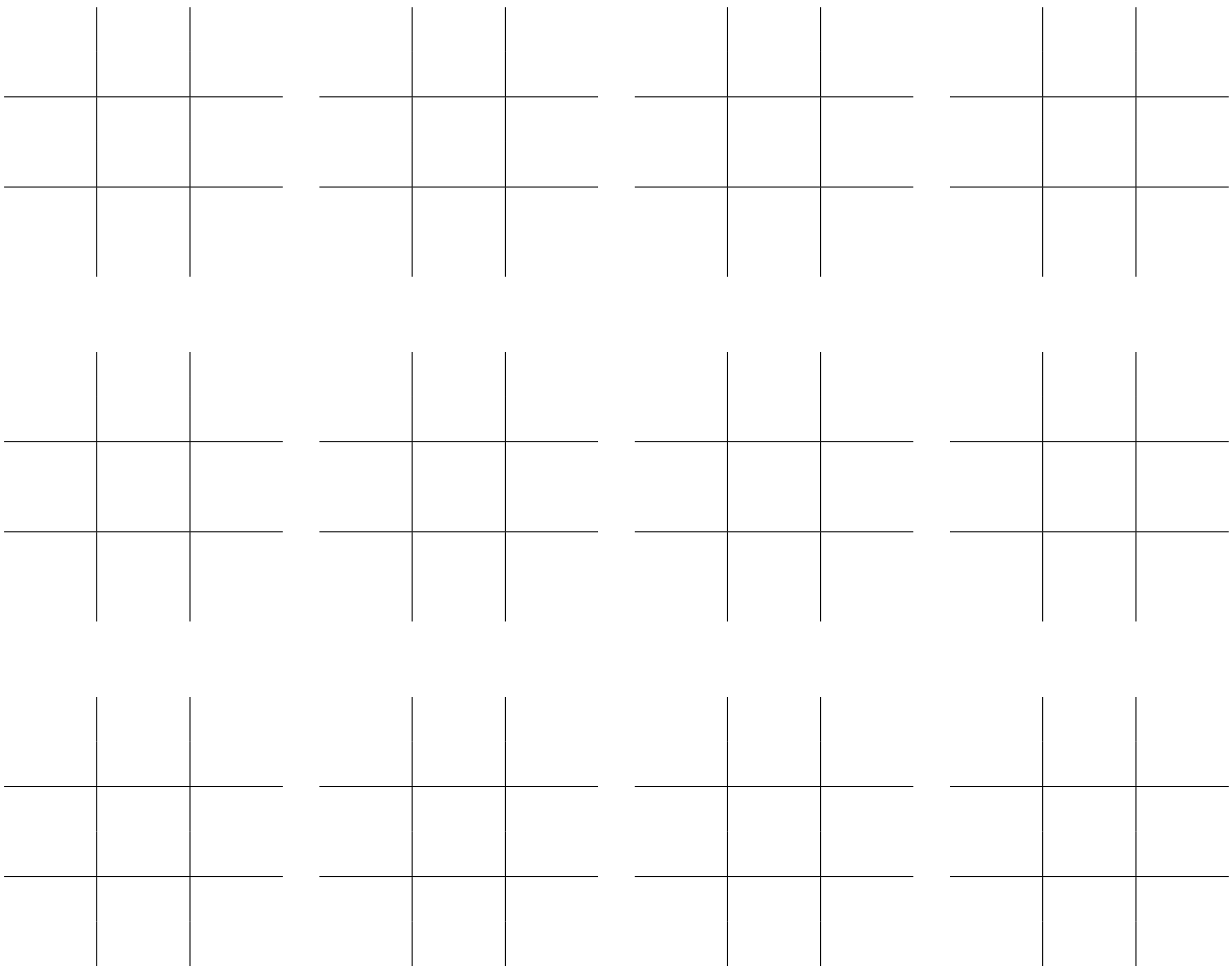
13. Without using a protractor, construct an angle that is equal to $30°$. (Hint: Use problems 2, 11.) Check the size of the angle with a protractor.

Chapter 4

Games

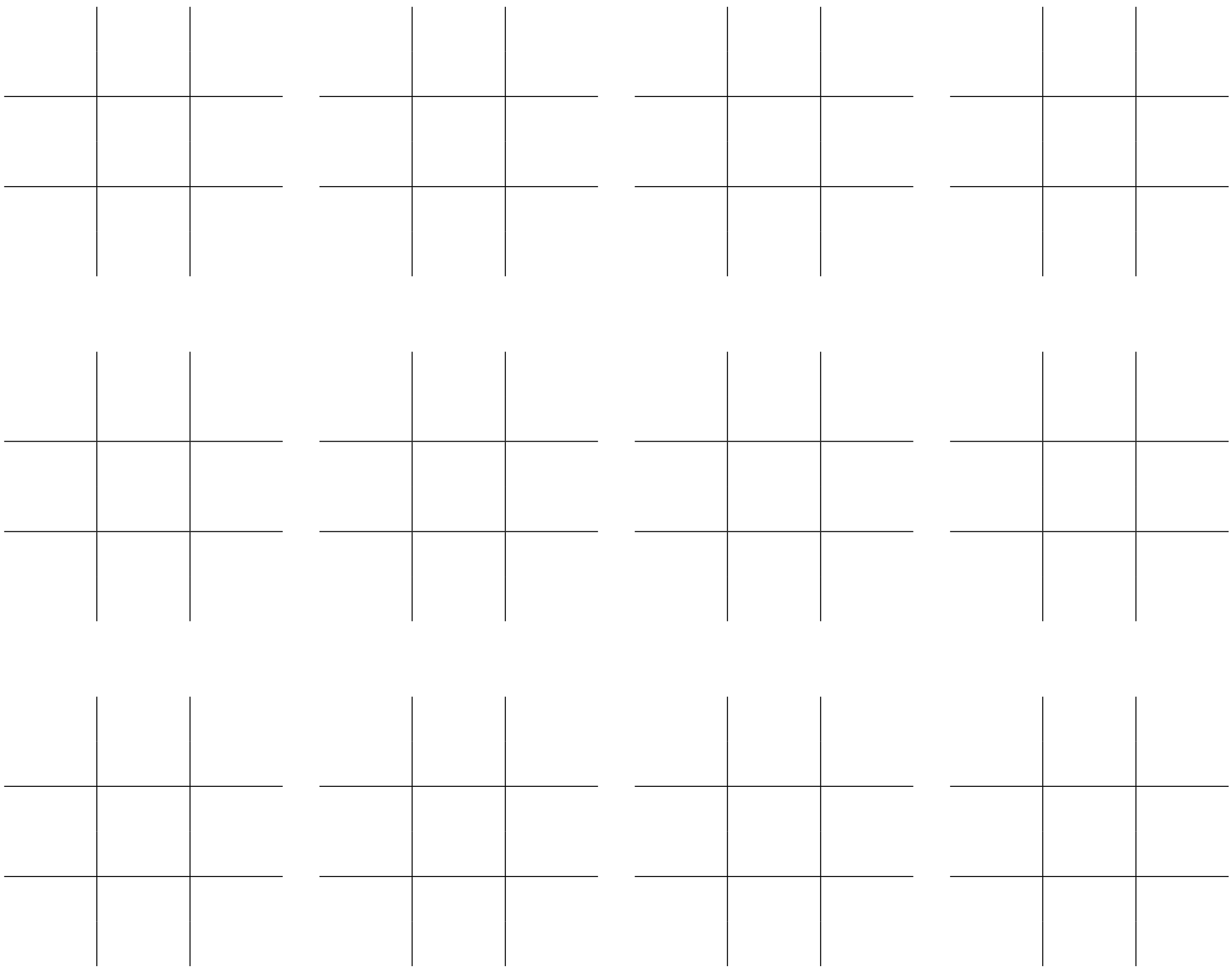
Mathematicians enjoy games. Games teach us to think, and thinking can be fun. Richard Guy describes the view that playing games cannot lead to serious mathematics as "puritanical" [4, p. xi]. In this chapter we present some games that do not involve chance and have no hidden information. Such games are known as combinatorial games.

Noughts and crosses This is an ancient game, usually known as tic-tac-toe in the US. Player A puts a nought in a cell. Player B then puts a cross in an empty cell. Continue. The winner is the player who can get three of their marks in a row (horizontally, vertically, or diagonally). If no player manages to do this during the game, then the result is a draw.



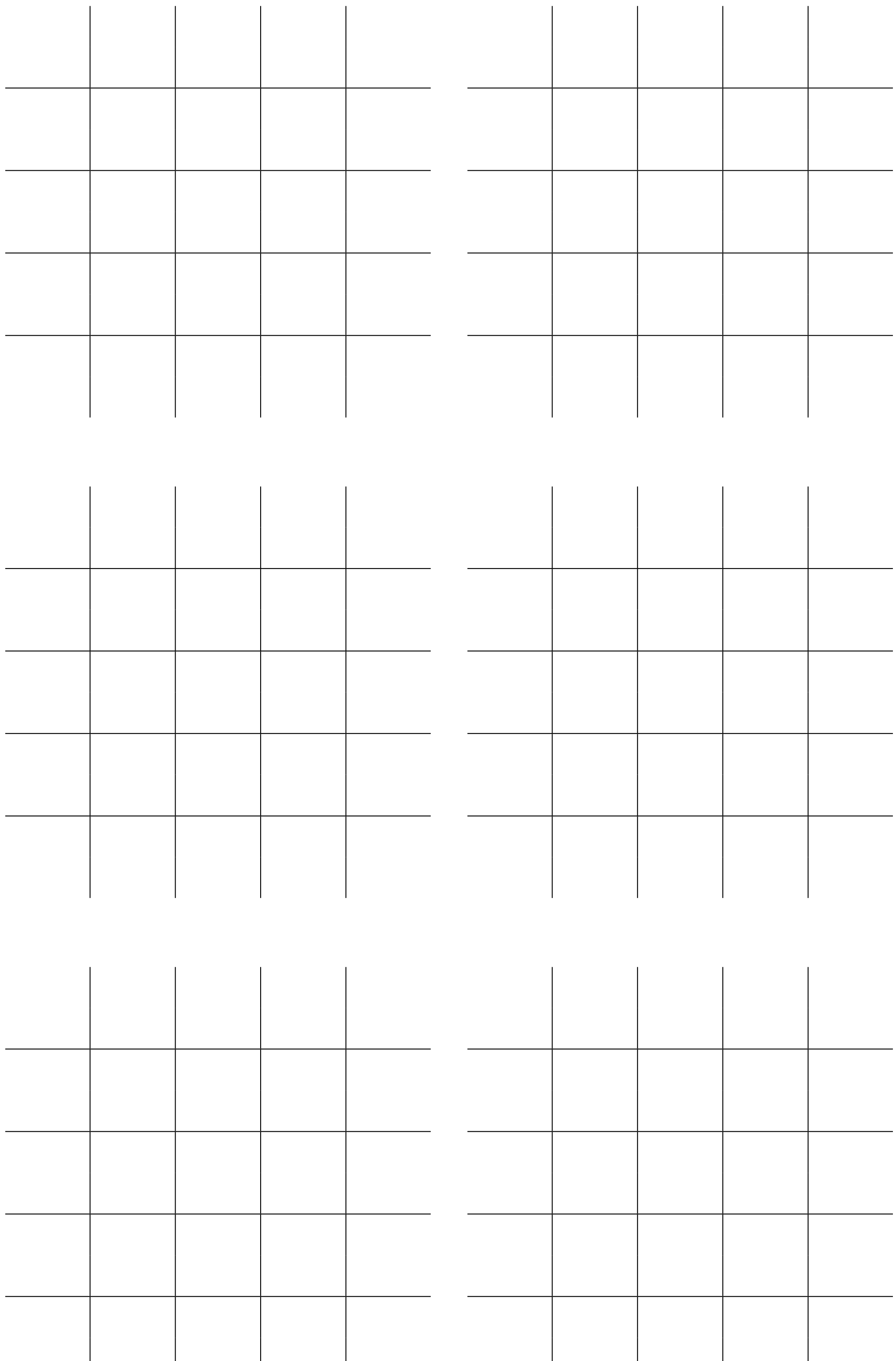
Is it an advantage to go first? Will going first guarantee a win? What is the best strategy? This simple game has many variants.

Noughts and crosses with a difference Player A puts a nought in a cell; player B then puts a cross in an empty cell. From then on, each player can decide whether to put a nought in an empty cell or a cross. The winner is the player who can get 3 of their marks in a row (horizontally, vertically, or diagonally). If no player manages to do this during the game, then the result is a draw.

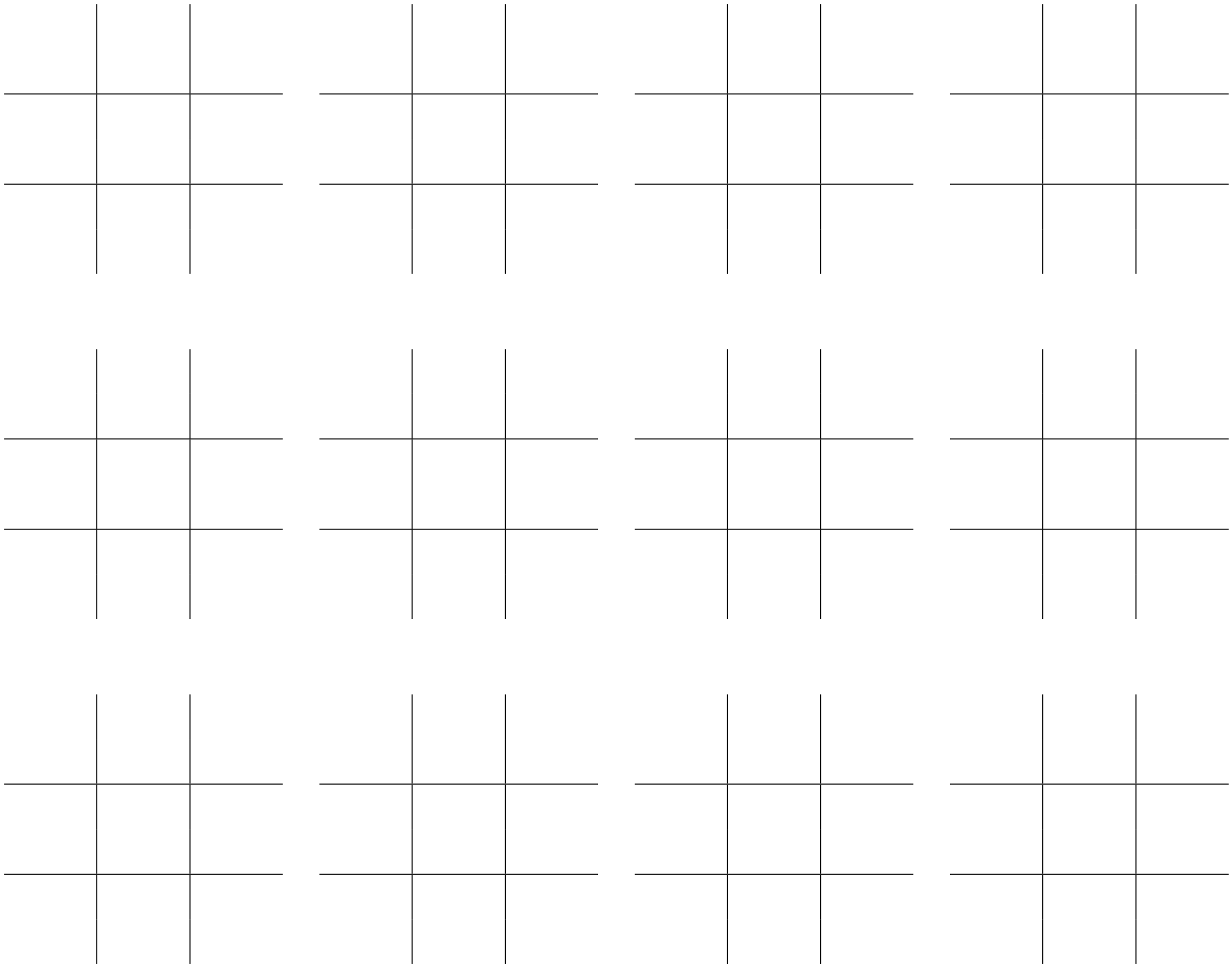


What is the best strategy?

5 × 5 noughts and crosses Now play noughts and crosses on a 5 × 5 grid. The winner is the player who can get 4 of their marks in a row [6].

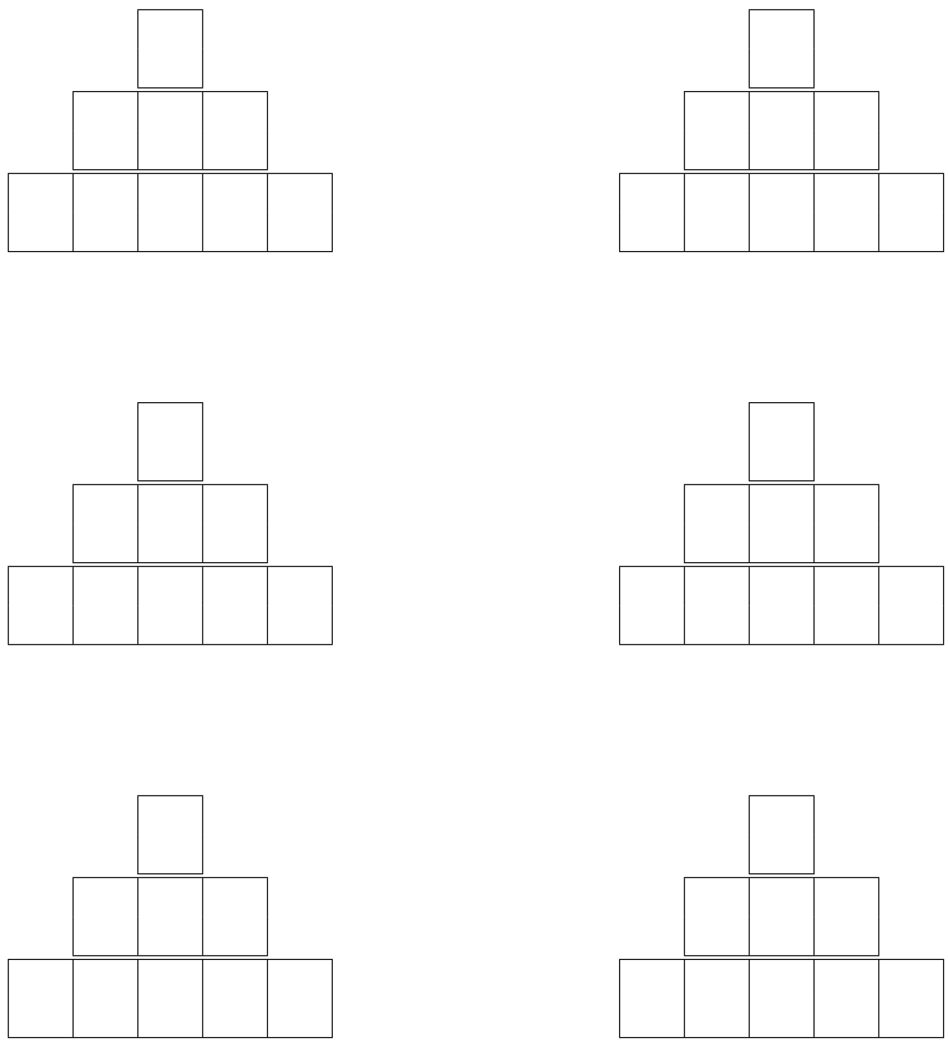


Losers noughts and crosses In this version, if you get three of your marks in a row, then you lose! If no player gets three in a row during the game, then the result is a draw [6].



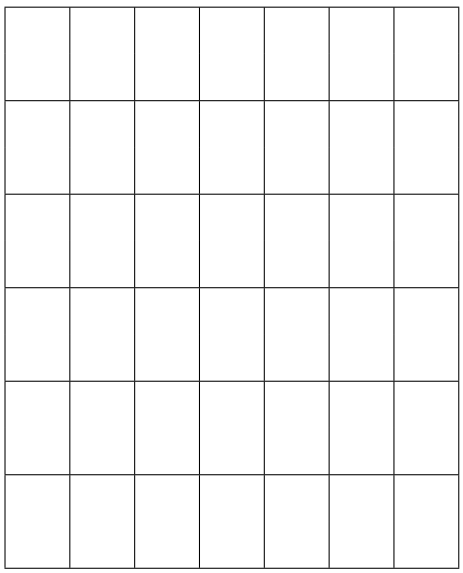
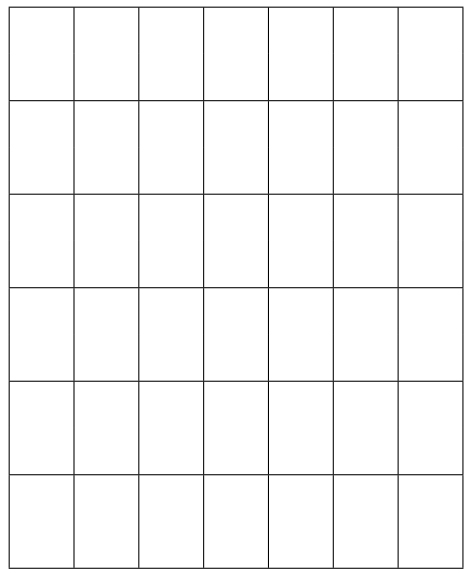
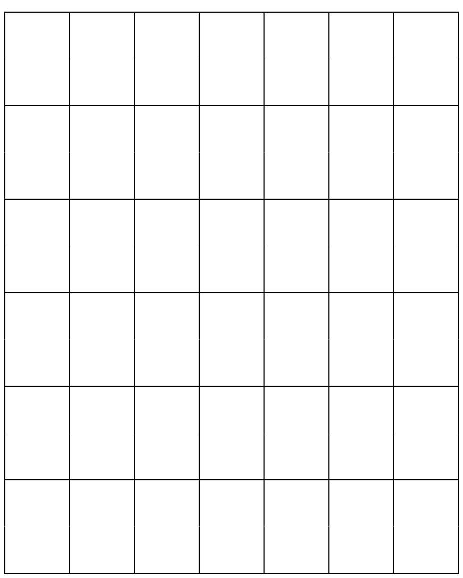
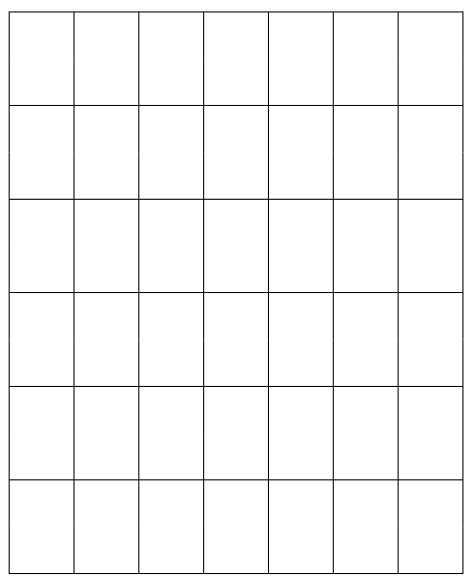
If you change the rules, then you change the way that people play the game. Is it an advantage to go first? Will going first guarantee a win? What is the best strategy?

Pyramid tic-tac-toe Same rules as tic-tac-toe, different shape board. Aim to get three marks in a row (horizontally, vertically, or diagonally). Enjoy. [6].



How many ways can you put three X's in a row (horizontally, vertically, or diagonally)?

A tic-tac-toe sort of game Play as in noughts and crosses with one major difference. Each player must place their mark in the lowest empty cell in a column. To win, a player must get four marks in a row (horizontally, vertically, or diagonally). If the board is full before this happens, the game is a draw [6].

Is it an advantage to go first? Will going first guarantee a win? What is the best strategy? These are mathematical questions!

Sprouts This game was invented by two mathematicians, John Conway and Michael Paterson. Sprouts is a game for two players. All you need is paper and a pencil. The game starts by drawing any number of spots. You might start with 3 spots. Each player in turn can make one of two moves.

- Draw a line that joins two spots and then place a new spot in the middle of that line

  OR

- Draw a line that joins one spot to itself, and then place a new spot in the middle of that line.

There are two restrictions.

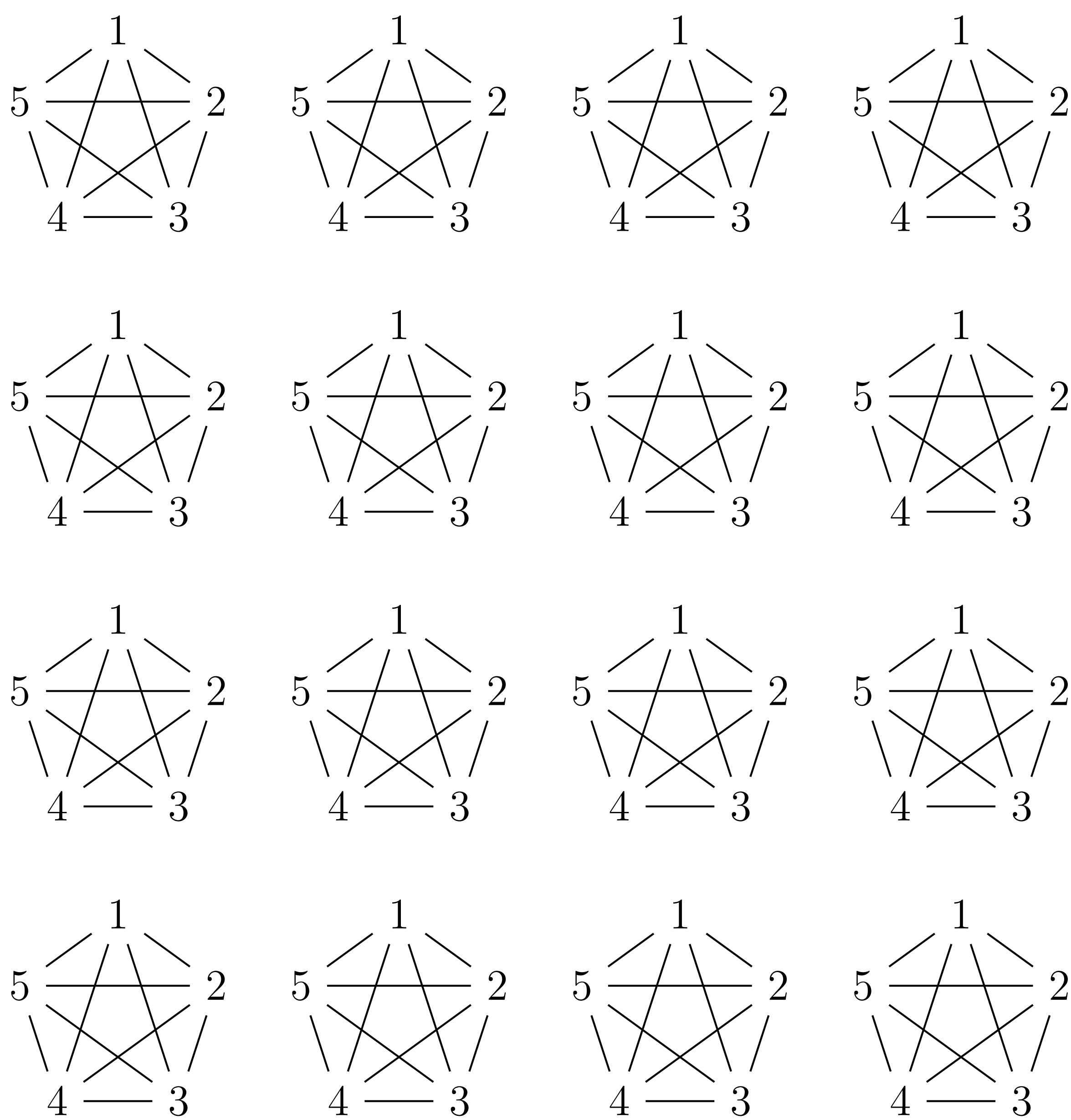
1. You are not allowed to draw a line that crosses another line.
2. No more than three lines can terminate at any spot.

The last person to move is the winner.

After playing this a few times, ask yourself, what are some good strategies in this game? Does the player with the first move have an advantage?

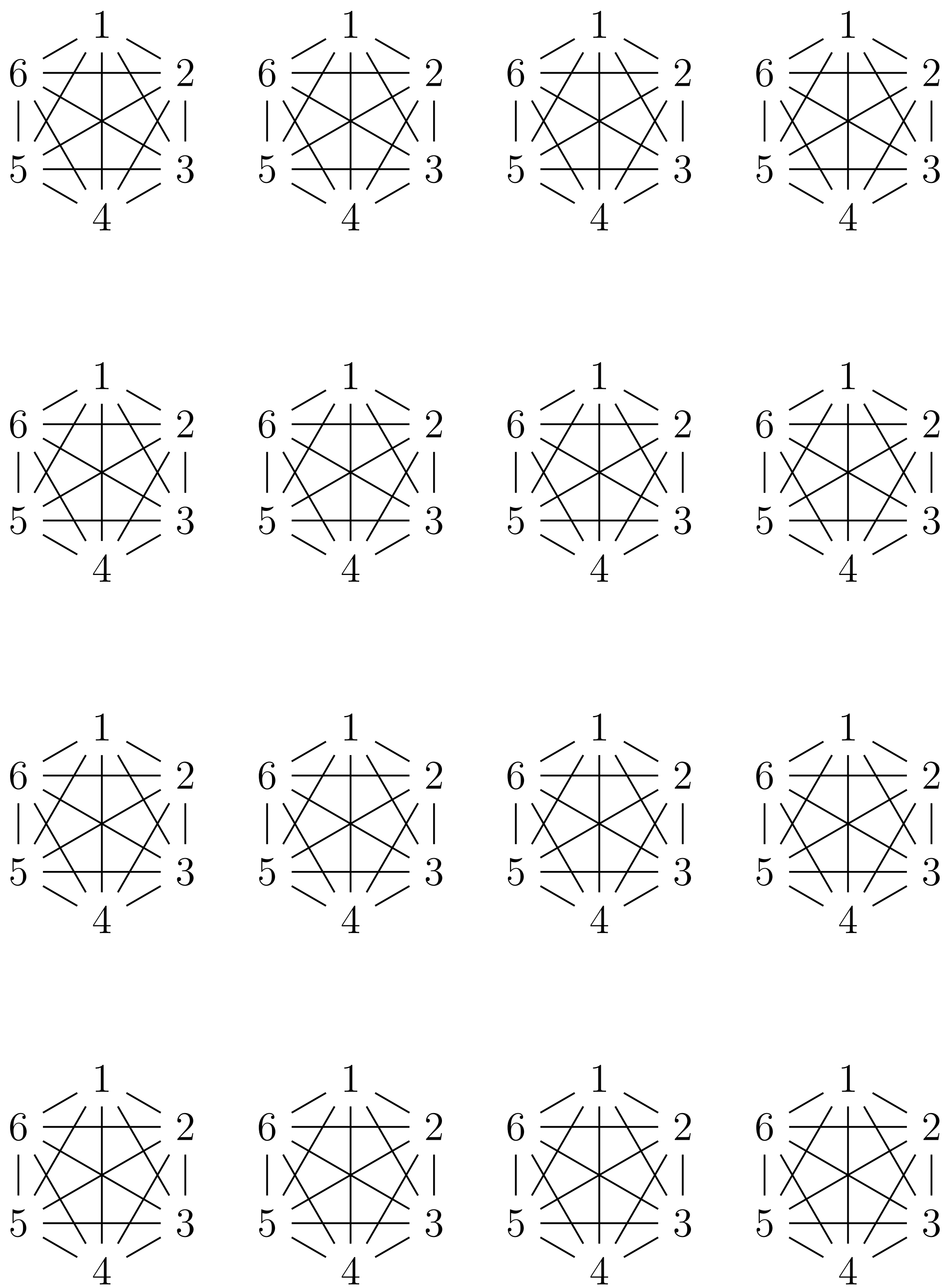
Playing Sprouts is a mathematical activity.

Colouring graphs A graph is a set of vertices connected by edges as in the diagrams below. The vertices are numbered and the edges join the numbered vertices. The players will have different coloured pens or pencils. Players take turns in colouring one of the uncoloured edges. The winner is the first person to colour edges that form a triangle (i.e. three lines connecting three vertices). If all the edges have been coloured without either player forming a triangle, then the game is a draw. Take turns in who goes first.



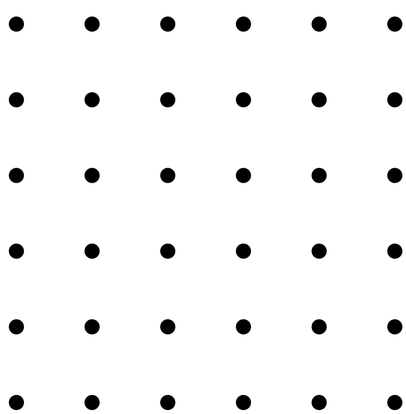
Believe it or not, colouring is a serious mathematical business. Is it an advantage to go first? Will going first guarantee a win? What is the best strategy? Want more?

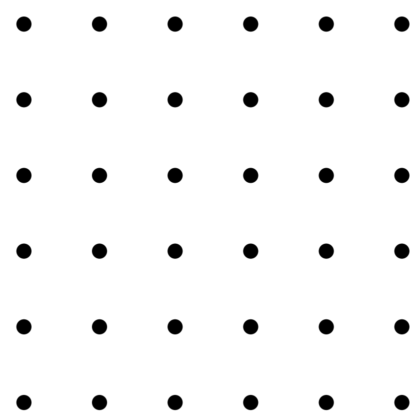
More colouring The winner is the first player to colour edges that form a triangle (i.e. three lines connecting three vertices). If there is no winner, then the game is a draw. Take turns in who goes first. This is mathematics.

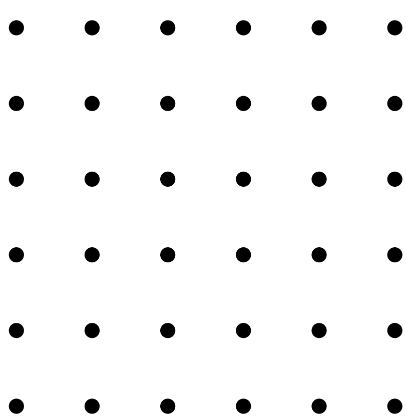


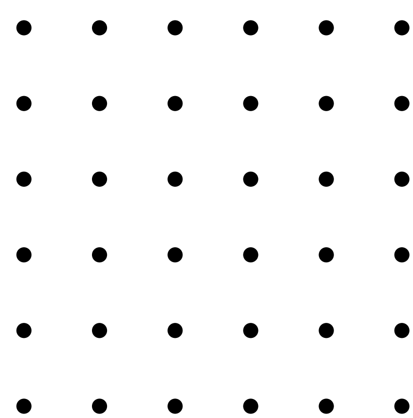
Paddocks Paddocks, often known as Dots-and-Boxes, was created in 1889 by Édouard Lucas, a French mathematician, who called the game "La Pipopipette". The game can be played played by two or more players.

The game starts with an empty grid of dots. Players take turns adding a single horizontal or vertical line between two unjoined adjacent dots. A player who completes the fourth side of a 1×1 box, claims the box, earns one point, and takes another turn. (A point is typically recorded by placing a mark that identifies the player in the box, such as an initial.) The game ends when no more lines can be placed. The winner is the player with the most points. Below are some grids for playing paddocks. Take it in turns to go first. What strategies did you use? Try it with more than two players. On these grids below, is it possible for the game to end in a draw?

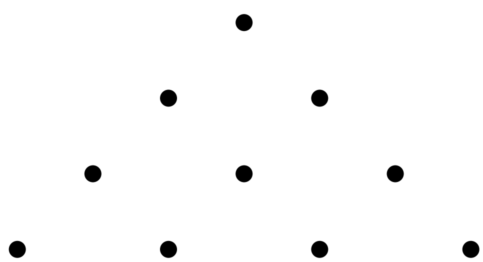
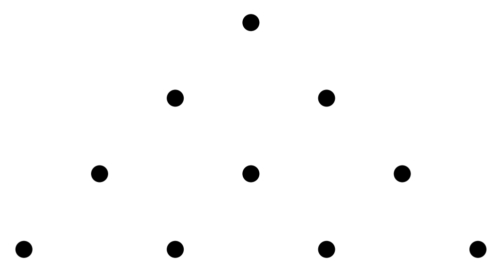
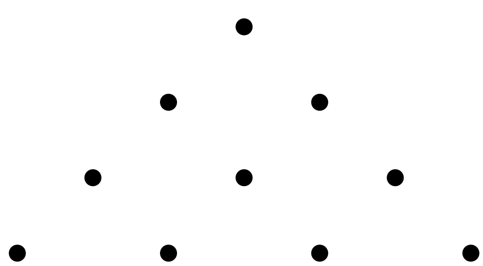
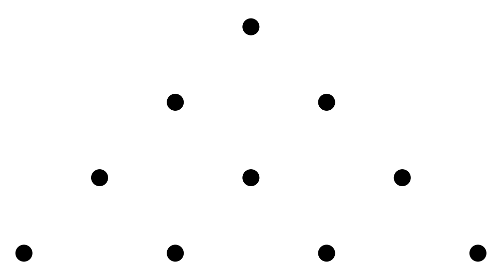
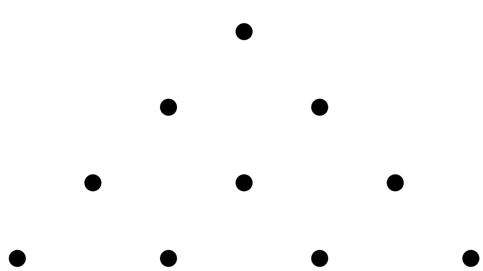
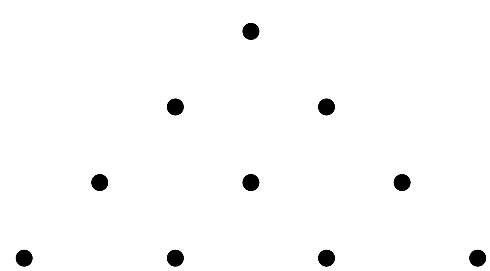








Triangular paddocks The game starts with an empty triangular grid of dots. Players take turns adding a single horizontal or vertical line between two unjoined adjacent dots. A player who completes the third side of a triangle, claims the triangle, earns one point, and takes another turn. (A point is typically recorded by placing a mark that identifies the player in the box, such as an initial.) The game ends when no more lines can be placed. The winner is the player with the most points. Below are some grids for playing triangular paddocks. Take it in turns to go first. What strategies did you use?

A traffic problem A city has a street grid as shown below. It has $8 \times 7 = 56$ corners. Show that you can make every street a one-way street so that it is still possible to get from any corner to any other corner.

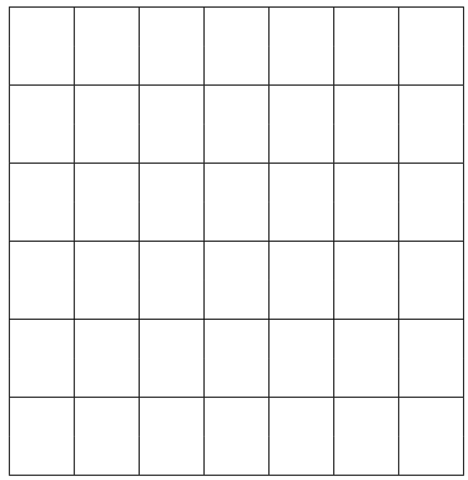


Figure 4.1: Rectangular street grid

Sudoku `https://www.edu-games.org/math/sudoku/sudoku-maker.php`

Nim Nim is a game of strategy in which two players take turns removing objects from distinct heaps of objects. On each turn, a player selects a heap and removes at least one object from that heap. The player who takes the last object loses.

A problem about relationships Suppose that we have only two sorts of relationships between any two people. Either they know each other, or they are strangers. We represent people as vertices on a graph. If two people know each other, then we link them with a blue edge. If they are strangers, then we link them by a red edge.

Suppose that three people are gathered together. Draw some graphs, with coloured edges, that illustrate their possible relationships. Is it true that, always, either all three people know each other or all three people are strangers to each other? (Answer: No)

Suppose that four people are gathered together. Draw some graphs, with coloured edges, that illustrate their possible relationships. Is it true that there must be a group of three who know each other, or there must be a group of three who are strangers to each other? (Answer: No)

Suppose that five people are gathered together. Draw some graphs, with coloured edges, that illustrate their possible relationships. Is it true that there must be a group of three who know each other, or there must be a group of three who are strangers to each other? (Answer: No)

Suppose that six people are gathered together. Draw some graphs, with coloured edges, that illustrate their possible relationships. Is it true that there must be a group of three who know each other, or there must be a group of three who are strangers to each other? (Answer: Yes)

Chapter 5

Projects

This section describes projects that are suitable for the secondary classroom. The ideal project will involve mathematics, problem solving and decision making, be able to be completed in class in several lessons, require little in the way of materials, and be related to scenarios with which Australian high school students will be familiar. Writing reports on these projects teaches us that there is more to mathematics than calculation! In these projects, you may need to use the internet.

Format of report

Each project should be submitted as a hand-written report with the following sections [8]. Take care with spelling and grammar. The length of the report should be 3–5 pages. Be proud of what you submit.

- **Title page:** Title, your full name, and date of submission. Choose a title that is short, and catchy. (Insert page numbers in the report.)
- **Introduction:** This states the problem that you will address in the report, and describes why the problem is important to you.
- **Methods:** This section describes how you went about answering the question and any materials that you used.
- **Results:** This section presents your results or findings.
- **Discussion:** This section contains a discussion of what the results mean to you. You might include things that were difficult, or other lines of investigation. You should list three things that you learnt from this project.
- **References:** The final section will be a list of references used including books, articles, and web-sites that you used.

Research problems

The following questions or problems can be the basis of your research projects. They are open-ended in order to give you the freedom to mould the project around the question. Be creative. Show all your working in your report. Keep a copy of your report.

1. Estimate the total cost of owning a car during the first three years of ownership.
2. Develop a comprehensive plan for going on a holiday for two weeks to the destination of your choice.
3. Develop a comprehensive plan for renovating a room in your house.
4. To choose a mobile phone provider that suits you, compare plans from two different companies.
5. The Australian Tax Office has produced several videos for students that explain various aspects of taxation. Describe and compare them:
`https://www.ato.gov.au/General/Education-zone/Videos-for-students/`
6. Estimate the cost of going to university or TAFE. (Choose your preferred course, and your preferred university or TAFE; include the cost of living and the cost of tuition even if you pay it off later - and that brings in the interest that has to be paid on the loan.)
7. Who was Pythagoras and what is Pythagoras' theorem?
8. Paddocks is a popular pencil and paper game. Play it several times with friends, sometimes you go first, sometimes you go second. Try various size "boards". Try playing with more than two players. In all this, take notes and think about your strategies. Describe your experiences. Which strategies seem to work, and which strategies do not seem to work?

Bibliography

[1] Child, J., Bertholle, L., & Beck, S. (2002). *Mastering the art of French cooking* vol. 1. London, UK: Penguin.

[2] Loyd, S. (1914/2007). *Sam Loyd's cyclopedia of 5,000 puzzles tricks & conundrums.* Bronx NY: Ishi Publishing.

[3] Feller, W. (1968). *An introduction to probability theory and its applications. Vol. 1, Third ed.* New York, NY: John Wiley & Sons, Inc.

[4] Guy, R. (Ed.) (1991). *Combinatorial games.* Prividence, RI: American Mathematical Society.

[5] Poundstone, W. (2012). *Are you smart enough to work at Google?* Oxford, UK: Oneworld Publications.

[6] Posamentier, A.S., & Krulik, S. (2017). *Strategy games to enhance problem-solving ability in mathematics.* Singapore: World Scientific.

[7] Sullivan, P., & Lilburn, P. (2017). *Open-ended maths activities: Using 'good' questions to enhance learning in mathematics.* South Melbourne, Australia: Oxford University Press.

[8] Perneger, T.V., & Hudelson, P.M. (2004). Writing a research article: Advice to beginners. *International Journal for Quality in Health Care*, *16*(3), pp. 191–192. Retrieved from `https://academic.oup.com/intqhc/article/16/3/191/1814554`

[9] Webster, M. (n.d.). Private communication.