

Estimating π down the ages

More than two thousand years ago, Archimedes discovered the famous equation that relates the circumference of a circle and the diameter: $C = \pi D$. Ever since then, mathematicians have explored many properties of the number π . We know today that $\pi = 3.141592654\dots$ In this lesson, we explore how π was approximated over the centuries.

Exercise 1: In the Bible, we find a description of the construction of Solomon's temple. The description contains the following passage. 'And he made the Sea of cast bronze, ten cubits from one brim to the other; it was completely round. Its height was five cubits, and a line of thirty cubits measured its circumference' (1 Kings 7:23, NKJV). From this description, sketch the bronze cast and find an estimate of π .

Formula of Madhava: Madhava of Sangamagrama (1350–1425) was a famous mathematician and astronomer from Kerala. 'It is only due to research into Keralese mathematics over the last twenty-five years that the remarkable contributions of Madhava have come to light.'¹ Madhava discovered the following formula for π .

$$\pi = \sqrt{12} \left(1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^3} + \frac{1}{9 \cdot 3^4} - \dots \right). \quad (1)$$

If you go far enough with equation (1) of Madhava, you can get very close to π .

Formula of Leibniz: In the 17th century, the famous German philosopher and mathematician Gottfried Leibniz (1646–1716) discovered another formula for π .

$$\pi = 4 \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots \right). \quad (2)$$

Leibniz proved that this infinite series also converges to π . If you go far enough with this formula (2), you can get very close to π . For example, we can find a poor approximation to π by using only the first four terms of Leibniz' formula:

$$4 \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \right) = 2.8952.$$

Exercise 2: Complete the table 1. I ask you to experience using the memory function on your calculator, and Excel in completing this exercise.

Exercise 3: Which formula is superior, that of Madhava or that of Leibniz? What does this suggest about the history of mathematics?

¹<http://www-history.mcs.st-andrews.ac.uk/Biographies/Madhava.html>