

2018 VCAA NHT Exam 2 Question 2 part e.:

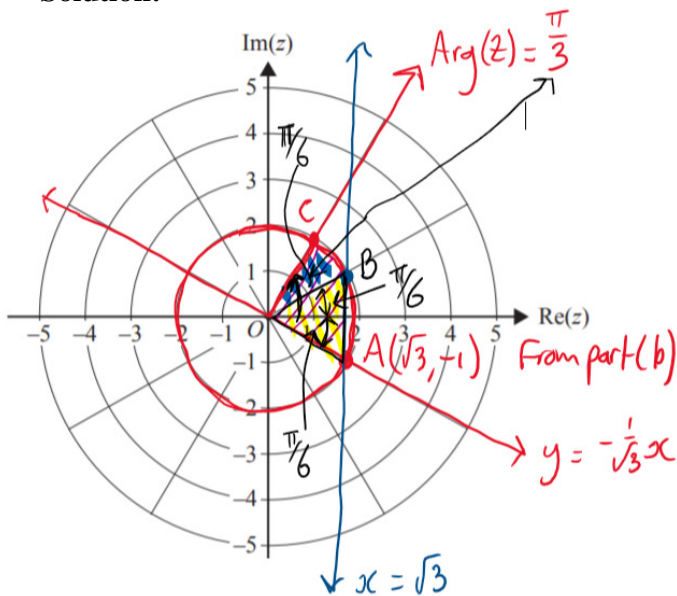
Question 2 (11 marks)

In the complex plane, L is the line given by $|z+1| = \left| z + \frac{1}{2} - \frac{\sqrt{3}}{2}i \right|$.

a. Show that the cartesian equation of L is given by $y = -\frac{1}{\sqrt{3}}x$. 2 marks

e. Find the area enclosed by L and the graphs of the relations $z\bar{z} = 4$, $\text{Arg}(z) = \frac{\pi}{3}$ and $\text{Re}(z) = \sqrt{3}$. 2 marks

Solution:



Area = Area of sector + Area of triangle OAB .

Angle subtended by sector = $\frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$:

Area of sector = $\left(\frac{\frac{\pi}{6}}{2\pi} \right)$ circle = $\left(\frac{1}{12} \right) \pi(2)^2 = \frac{\pi}{3}$.
Fraction of area of circle

$\angle BOA = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$ and $OB = OA = r = 2$:

Area of triangle $BOA = \frac{1}{2}(2)(2) \sin\left(\frac{\pi}{3}\right) = \sqrt{3}$
From the VCAA formula sheet

Therefore area = $\frac{\pi}{3} + \sqrt{3}$.