Irrational numbers: Assignment

Date due: 31 March 2022

- Please do this assignment in your workbook.

- You might map out a draft answer to a question before putting the final copy into your workbook. You may work in pencil if you wish.

- The quality of your explanations and the quality of your mathematics are both important. Writing is important in mathematics!

- Fair collaboration is encouraged.

- Total marks = 100

You could spend your whole life learning about irrational numbers. There are still unanswered questions about such numbers. The learning intention of this assignment is use your knowledge of indices and radicals to to begin exploring the world of rational and irrational numbers. You will succeed in this assignment if you submit it on time and say to yourself "I did my best—and I learned something from the experience".

1. Explain why the following numbers are rational numbers.
$$\frac{22}{7}, \quad 4, \quad \frac{-5}{10}, \quad 1.2, \quad 0.\dot{3}. \quad \textbf{(10 marks)}$$

2. Prove that the sum of two rational numbers is always a rational number. [Hint: Show that $2/3 + 4/5$ is rational; then generalise to $m/n + p/q$.] **(10 marks)**

3. Give an example to prove that it is possible for the sum of two irrational numbers to be a rational number. [Hint: You may use the fact that $\sqrt{2}$ is irrational.] **(10 marks)**

4. Prove that $\sqrt{3}$ is an irrational number. [Hint: Imitate the proof for $\sqrt{2}$.] **(10 marks)**

5. Prove that the average of two rational numbers is always a rational number. **(10 marks)**

6. You have just proved that the average of two rational numbers must be a rational number. Use this repeatedly to prove that, between any two given rational numbers, there are infinitely many rational numbers. **(10 marks)**

7. Give an example to prove that it is possible for the average of two irrational numbers to be a rational number. [Hint: Again, you may use the fact that $\sqrt{2}$ is irrational.] **(10 marks)**

8. If we assume that π is an irrational number, prove that $\pi + 5$ is also an irrational number.

   [Hint: Assume that $\pi + 5$ is rational and see where it leads you.] **(10 marks)**

9. The number $\phi = \dfrac{1 + \sqrt{5}}{2}$ is a famous irrational number known as the golden ratio. Prove that $\phi^2 = \phi + 1$. [Hint: Simplify ϕ^2, then simplify $\phi + 1$, and observe that they are equal.] **(10 marks)**

10. What did you learn from this assignment that you did not know before? **(10 marks)**

Reference

Hassani, M., & Mills, T. (2015). Beyond the rational. In D. Martin et al. (Eds.) *Back to the future*, Proc. 52nd Annual Conf. Math. Assoc. Vic., 3–4 Dec. 2015. Brunswick: MAV, pp. 54–60