Knights and Knaves

The study of logic dates back at least to the time of Aristotle (384 BCE–322 BCE) in ancient Greece. Logic is now an important part of mathematics, and essential to other disciplines such as computer science, the law, economics. Indeed, logic will help you whenever you want to develop an argument.

Learning intention and success criteria

The intention of this lesson is to learn some of the basics about logical reasoning through problem solving. Knights-and-knaves problems are quite famous in the world of logic. They may not look mathematical, but they are, because logic is part of mathematics. And they are fun. You will be successful if you solve some of the problems below—**and can explain your reasoning**. Then you will have a better understanding of proof, and proof is the glue that holds mathematics together.

Problems

Strategy: Choose a statement; e.g. Alice is a knight. Assume it is true, and see where it leads. If the assumption leads to a contradiction, it must be wrong: choose another statement.

1. *On an island there are only knights and knaves. Knights always tell the truth; knaves always lie.*

   (a) Alice and Bob are inhabitants of the island of knights and knaves. Alice makes the following statement: "At least one of us is a knave." What are they actually?

   (b) On the island of knights and knaves, you are approached by two inhabitants, Alice and Bob. Alice said to you, "We are both knaves." What are they actually?

   (c) On the island of knights and knaves, you were approached by two people, Alice and Bob. Alice said, "Bob is a knave." Bob then said, "Neither of us is a knave." What are they actually?

   (d) Three of the inhabitants (Alice, Bob, and Chris) were standing together in a garden. A stranger passed by and asked Alice, "Are you a knight or a knave?" Alice answered, but rather indistinctly, so the stranger could not make out what she said. The stranger then asked Bob, "What did Alice say?" Bob replied, "Alice said that she is a knave." At this point Chris, said, "Don't believe Bob; he is lying!" Is it possible that Alice said "I am a knave" as claimed by Bob? What are Bob and Chris? What about Alice?

   (e) Suppose the stranger asked Alice, "What is the exact number of knights among you?" Again Alice mumbled indistinctly. So the stranger asked Bob, "What did Alice say?" Bob replied, "One." Then Chris said, "Don't believe Bob; he is lying!" Now what are Bob and Chris? What about Alice?

   (f) On the island of knights and knaves, you were approached by three people, Alice, Bob, and Chris. Alice said, "At least one of the following is true: either Chris is a knave or I am a knight." Bob said, "Alice could claim that I am a knave." Chris says, "Neither Alice nor Bob are knights." Who is a knight and who is a knave?

   (g) What is a knave? (Hint: The Queen of Hearts: she made some tarts et cetera)

2. *On another island, there are three types of people: knights, who always tell the truth; knaves, who always lie; and spies, who can lie or tell the truth.*

   (a) On the island of knights and knaves and spies, you are approached by three inhabitants, Blue, Red, and Green. You know that one is a knight, one is a knave, and one is a spy. They speak in the following order: Blue says, "I am a knight." Red says, "He speaks the truth." Green says, "I am a spy." Who is the knight, who is the knave, and who is the spy?

   (b) On the island of knights and knaves and spies, you come across three inhabitants Blue, Red, and Green. You know that one is a knight, one is a knave, and one is a spy. Blue says, "Red is the spy." Red says, "Green is the spy." Green says, "Red is the spy." Who is the spy? Who is the knight and who is the knave?

   (c) On the island of knights and knaves and spies, you are approached by three inhabitants, Blue, Red, and Green. You know that one is a knight, one is a knave, and one is a spy. Blue says, "I am not a spy." Red says, "I am a knave." Green says, "If you asked me, I would say that Red is the spy." What are the true identities of these inhabitants?

   (d) Does Australia have spies?

Reflection

What did you learn from this lesson?

Sources

Raymond Smullyan was a well-known mathematician and logician. Some of the problems above are based on his book entitled *What is the name of this book?*

Some of the problems appeared in several issues of the magazine *Popular Mechanics*.
See `https://www.popularmechanics.com/riddles-logic-puzzles/`