

Unsanctioned Corrections to VCAA Implementation Material – Proof by Contradiction

Example 3: Continuation from $e^a = 5^b$. (*)

Now 5^b is an integer therefore e^a must be an integer and so $e^a - k = 0$ for some $k \in \mathbb{Z}^+$.

But e is a transcendental number and so, by definition, there is no integer polynomial with finite terms that has e as a root (**).

Therefore we have a contradiction in the fact that e is transcendental.

Therefore $\log_e(5)$ is irrational.

* After the line $e^a = 5^b$, it is **not** correct to immediately conclude a contradiction by making a casual remark such as:

“It's clear at this point that the right hand side must be divisible by five but the left hand side is not.”

VCAA support material for implementation of the VCE Mathematics Study Design (2023-2027) in schools:
Proof by Contradiction transcript.

Such remarks essentially assume the conjecture that we are attempting to prove.

** This requires students to know a basic fact about transcendental numbers, which is arguably outside the scope of the VCE Study Design. In which case, the VCAA Example 3 is poorly chosen. Proving the irrationality of $\log_{10}(2)$ would be far more appropriate.