Unsanctioned Corrections to VCAA Implementation Material – Proof by Induction

Example 1: Prove by mathematical induction that $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}=1-\frac{1}{2^n},\ \ n\in N$.

Solution:

Let $P(n)$ be the conjecture $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}=1-\frac{1}{2^n},\ \ n\in N$.

Base case:

Show $P(n)$ is true for a particular value of *n*.

Try $n=1$: It is required to prove that $\frac{1}{2}=1-\frac{1}{2^1}$.

Left Hand Side $=1-\frac{1}{2^1}=\frac{1}{2}=$ Right Hand Side.

Therefore $P(1)$ is true.

Inductive hypothesis:

ASSUME $P(n)$ is true for some $n=k\in N$, that is, assume $P(k)$:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^k}=1-\frac{1}{2^k}.$$

Show that $P(k)\Rightarrow P(k+1)$:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^k}+\frac{1}{2^{k+1}} = \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^k}\right)+\frac{1}{2^{k+1}}$$

$$=\left(1-\frac{1}{2^k}\right)+\frac{1}{2^{k+1}} \quad \text{using the inductive hypothesis}$$

$$=1-\frac{2}{2^{k+1}}+\frac{1}{2^{k+1}} \qquad =1+\frac{1-2}{2^{k+1}} \qquad =1-\frac{1}{2^{k+1}}$$

which is $P(k+1)$.

Since $P(1)$ is true and $P(k)\Rightarrow P(k+1)$, it follows from induction that $P(n)$ is true for $n\in N$.