

Probability: A German life table

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Life tables are useful in many areas such as public health, economics, and insurance. In the following problems we will see how life tables contain fundamental ideas in probability, and tell us something about the society on which the table is based.

Learning intention and success criteria

The purpose of these problems is to show you how life tables can be used to improve our understanding of probability. You will be successful in this lesson if you solve a good number of the problems below. To be successful in solving these problems, talk to your colleagues, ask questions, say “I’m stuck” when you are stuck.

Problems

The data in Table 1 are based on Life Tables, 2019, Germany published by the World Health Organisation (WHO) in 2020. These data are based on mortality estimates and can be read as follows. Imagine that we start with 100,000 males aged 0. By age 5, only 99673 of them have survived; by age 10, only 99612 have survived; and finally only 38675 survive to age 90. One reads the data for females similarly.

Calculate the following probabilities correct to 4 decimal places. Show your working.

1. What is the probability that a German female survives until age 5?
2. What is the probability that a German male survives until age 5?
3. Why might there be such a difference between the answers to questions 1 and 2?
4. What is the probability that a German female survives to age 80?
5. What is the probability that a German male survives to age 80?
6. What is the probability that a German female does not survive to age 80?
7. What is the probability that a German male does not survive to age 80?
8. Suppose that 100 German male children were born in the same year. How many would you expect to survive to age 80?
9. Suppose that 100 German female children were born in the same year. How many would you expect to survive to age 80?
10. Suppose that 50 German children were born in the same year: 30 of them were male, and 20 were female. How many children would you expect to survive to age 80?

11. Estimate the age by which 25% of the population of 100,000 German males have died. Estimate the age by which 25% of the population of 100,000 German females have died. Explain how you made these estimates.
12. This problem is related to life expectancy. Estimate the age by which 50% of the population of 100,000 German males have died. Estimate the age by which 50% of the population of 100,000 German females have died. Explain how you made these estimates.
13. This problem deals with conditional probability. The probability that a female German aged 20 will survive until age 60 is $96712/99599 = 0.9710$. Explain this calculation.
14. What is the probability that a German male aged 20 will survive until age 60?
15. Suppose that a German woman is aged 20 and a German man is aged 20. What is the probability that both will survive until aged 60?
16. Three randomly chosen German men are aged 20. What is the probability that all of them will survive until aged 60?
17. Three randomly chosen German women are aged 20. What is the probability that not all of them will survive until aged 60?
18. What general impressions do these calculations make about the difference in the lives of males and females in Germany? Give reasons for your impressions.

Age	Males	Females	Age	Males	Females
0	100000	100000	55	96288	97814
5	99673	99724	60	94307	96712
10	99612	99673	65	91010	94862
15	99571	99638	70	85683	91863
20	99527	99599	75	78247	87540
25	99381	99522	80	68301	80936
30	99170	99440	85	55487	71865
35	98946	99332	90	38675	56384
40	98628	99155			
45	98156	98899			
50	97439	98502			

Table 1: Data extracted from Life Tables, Germany, 2019 (WHO)

Reflection

Looking back over your work, what did you learn from this set of exercises?